

# Affordance and symmetry in user interfaces

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Affordance is a widely-used term in human-computer interaction that, while familiar and attractive, does not have a clear operational definition. Using the mathematical concept of symmetry, this paper shows it is possible to begin developing an operational definition for significant aspects of affordance by forming the theoretical concept of symmetry-affordance. The proposed definition restricts symmetry-affordance to particular contexts but in doing so makes it more useful, as it is clear how to exploit symmetry to aid design. The definition is in standard mathematics (in fact, group theory and model theory) and requires little additional structure. In examining symmetry-affordance, it becomes clear that some other HCI notions can be similarly interpreted by symmetry. The paper provides examples and design insights.

“Symmetry, as wide or as narrow as you may define its meaning, is one idea by which man through the ages has tried to comprehend and create order, beauty, and perfection.”

*Hermann Weyl* [1]

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## 1. TERMINOLOGY IN THE FIELD OF HUMAN COMPUTER INTERACTION

As disciplines develop, they develop their own terms that capture ideas unique to the discipline, that aid brief, concise communication and that, as the mathematician Whitehead pointed out, can actually lead to more insightful reasoning [2]. Accordingly, the field of interaction design (usability, Human Computer Interaction (HCI), human factors in computing) has developed terms such as affordance, mode, consistency, flow, user model and so on. These terms however tend to suffer from an evolution that seems to expand their meaning or expand the contexts in which they can be used but often to the point where they become of little real value.

The most famous example of this is Norman’s adoption of the term affordance to describe elements of interaction design [3], and Gaver’s generalisation of it to desktop GUI interfaces [4]. Whilst initially “affordance” captured a particular aspect of making user interfaces usable, it has been subsequently adopted, stretched and adapted to the point that it has very little substantial meaning; Hartson [5] gives a good review; Oshlyansky *et al.* [6] discusses the cultural aspects of affordance; and Brown and Duguid [7] argue affordance extends even to social practices.

The development and extension of the affordance concept reached its culmination in Norman’s rejection of it as a useful concept [8]. Affordance is now really only used in hindsight; it means what we want it to mean in order to draw attention to some existing design issue that is sort-of affordance. Although this is useful, we believe we can do better by having a clearer idea of what a specific concept of affordance entails.

The effect of terms becoming elastic then being over-stretched is that they become vague and, in fact, useless. In general, the knock-on effect is that there is a constant shifting in terminology in order to capture new terms that have better communicative value. The culture of rapid change in the field’s emphasis and terminology is no more evident than in the constant re-branding of the discipline itself: as, for example, illustrated by the annually changing themes of the BCS HCI conference.

The goal of this paper is to provide a more operational definition for affordance — that is the goal; in fact, affordance is a large and complex topic, and we can only start on the route. The key to our approach is that aspects of affordance can be understood in terms of symmetries. Symmetry is a wide-reaching yet nonetheless explicit mathematical concept. This means that those aspects of affordance captured by symmetry cannot shift in their meaning. Along the way, it becomes clear that symmetry also captures aspects of

the notions of consistency and mode that have shown an equally elastic expansion in meaning.

Symmetry in its common and naïve form is a simple concept, however in this paper we take a more general abstract approach to symmetry — indeed, we make some contributions to a general theory of symmetry. The central part of the paper is a formalisation of symmetry as applied to HCI; specifically, we present an initial formalisation of key parts of affordance, and thus introduce a precise concept: *symmetry-affordance*. This provides a more flexible and broad-reaching approach to understanding particular sorts of affordances without losing rigour, and hence the concrete concepts behind the types of affordance that we discuss. This paper, then, makes a theoretical contribution to the research field; though this abstract approach may be beyond the mathematical experience of some designers and practitioners within the field of HCI, the insights and new questions that it yields are very concrete and clear. These are summarised at the end of the paper.

## 2. AFFORDANCE

The user interaction issues addressed by the modern concept of affordance have been recognised for a long time (e.g., Murrell’s classic 1965 book on ergonomics [9] discusses control “compatibility” and the issues of population stereotypes extensively). A concept that is similar to affordance in its application to user interface design but predates it is *passivity* [10]. The term affordance itself was introduced by Gibson [11] when he tried to break out of a narrow “laboratory” view of psychological vision research to understanding real-world vision. In the laboratory, we may be interested in how neurons respond to straight line stimuli; in the real world, however, we need to recognise complex visual stimuli, such as staircases, and respond appropriately to them. We see staircases in real life from many points of view, and we can recognise how to use them (e.g., to climb them) despite wildly varying retinal images. Whatever our neurons are doing to retinal images of lines, something else of significance is also happening.

Gibson coined the word affordance to mean the interactive properties of an object recognised as such from the stimulus (often visual). Although he used phrases like the “brain resonates” with the object, he also used more formal words like “invariance.” Somehow, he claimed, what an object affords is invariant under visual transformations. Thus a staircase affords climbing, whether we see it straight on or from almost any angle. Even spiral staircases afford climbing. Despite all the various possible optical images a staircase might have on our retinas, the same concept (here, climbing) is invariantly perceived.

Definitions of affordance in standard HCI textbooks express affordance straight-forwardly, if vaguely, as a desirable concept for effective user interface design [quoted below with their original emphases]:

- “An affordance is an aspect of an object that makes it obvious how the object is to be used. In user interface displays, the features that create affordances are usually visual . . .” [12, p124]
- “In designing to accommodate visibility, each function and the method of operating it would be apparent — to most people in the culture for which it is intended — merely by looking at it. A control that has this attribute has come to be called an **affordance**.” [13, p63]
- “The psychological idea of *affordance* says that things may suggest by their shape and other attributes what you can do with them: a handle affords pulling or lifting, a button affords pushing.” [14, p135] . . . “Some psychologists argue that there are intrinsic properties, or affordances, of any visual object that suggest to us how they can be manipulated.” [14, p166]

It is all very well that “a button affords pushing,” but can we use the concept of affordance to help critique and design new user interfaces, or new user interface objects? Even button pressing has its problems, as Norman points out [8]: everything on a screen is “just” a picture, so somehow the user has to learn that the pictures of buttons are merely representations of objects that can be clicked on — and even that is a complex concept since a mouse button, not the screen, is pressed (unless the screen is a touch screen).

While Gibson’s ideas do not explain anything about the mechanism of vision or perception in general (in fact, he denied internal representations), they attempted to lift vision research from working out how things are implemented by neurons to what vision achieves: in David Marr’s terms, affordance as a research topic removes implementation bias [15]. Ironically, affordance in user interface design has become entwined with implementation-specific examples, like debating buttons and their representations as pictures on screens!

Norman’s insight was that if we are designing objects (say, stairs) then we can design them so that how to use them is more obvious, namely affordance is here understood as a constructive design goal, not merely as a question for psychological inquiry. If, then, affordance is a constructive design idea, we surely want to use it constructively rather than as a *post facto* criterion? It is of course useful to evaluate user interfaces in an iterative design cycle: optimising features or behaviour that we agree to call affordance in subsequent design iterations is worthwhile, but in an ideal world design principles would be deployed earlier than evaluation is possible. (Indeed, in industry, by the time significant evaluation is possible, the product is already shipping and scope for improvement is reduced.) Thus in formalising affordance we will want to isolate from it what principles it can contribute to “abstract,” viz pre-implementation, design; and then it can be wielded constructively.

### 3. SYMMETRY

Symmetry is a powerful notion in a variety of ways, as this section of the paper now reviews. Weyl [1] is the classic reference on symmetry; Stewart [16] is more recent and provides a historical perspective on the mathematics that is largely implicit in Weyl.

Most obviously, many visual symmetries leap out at us without prompting. For example, the lateral symmetry of faces and bodies, the rotational symmetries of geometric figures and even the enormously complex symmetries of Islamic art occur to us, apparently, immediately on seeing them. Weyl [1] is the classic reference.

There are various theories about why symmetric faces are more attractive. Illness (such as chicken pox) and physical injuries have no reason to respect the body's symmetries, and they therefore tend to leave asymmetric scars. Asymmetry is thus indicative of possible weaknesses. Another theory is that symmetry requires fitness in order to be able to co-ordinate actions that are spatially separate. The symmetry of a face therefore indicates the fitness of the mate.

A less evolutionary approach to explain the appeal of symmetric faces is simply that the information processing cost is less: a laterally symmetric object is quicker to recognise because literally only half the object needs to be processed. Empirical studies have shown that faces that are more symmetric are rated as more attractive simply because of this cognitive fluency [17]. This theory also extends to other symmetric objects and may explain why we find them generally more attractive than asymmetric objects.

Learning to survive is also important. In the natural world, if we (or others) are attacked (say, by a lion) we do well to generalise the lessons learnt on handling the attack. We would learn very slowly if when we are attacked from the left, we only learnt how to respond to attacks from the same direction! However, the symmetry of the physical world justifies learning how to handle an attack from *any* direction when we only have experience of attacks from one direction.

As well as survival value and aesthetic appeal, symmetry also has a logical appeal. Knowing that something is symmetric means we can abridge a description of it, so just as a symmetric face requires less attention, so too do more abstract symmetries.

For example, in combinatorics, when we choose  $k$  things out of  $n$ , we are effectively choosing the  $n - k$  things to leave behind, so

$$\binom{n}{k} = \binom{n}{n-k}$$

This symmetry identity can also be seen immediately from the symmetry of Pascal's triangle where the  $k^{\text{th}}$  number in the  $n^{\text{th}}$  row is simply  $\binom{n}{k}$ :

$$\begin{array}{cccc} & & & 1 \\ & & & 1 & 1 \\ & & & 1 & 2 & 1 \\ & & & 1 & 3 & 3 & 1 \\ & & & 1 & 4 & 6 & 4 & 1 \\ & \dots & & & & & & \dots \end{array}$$

With symmetric functions like  $x^2$ , it is first straightforward to recognise that any such quadratic function has two roots but then further recognise, that once we know one root, we know the other by symmetry. For example, we may calculate (perhaps with some effort) that  $x^2 = 5$  has a root at 2.236067977, then we know without any further effort that there is another root at  $-2.236067977$ . The elementary trick of "completing the square" merely transforms a general quadratic so that it is symmetric about zero, and hence much easier to solve.

The use of symmetry in mathematics reaches its abstract apogee in proofs where it is not unusual to have an argument along the following lines:

- (i) Label two objects arbitrarily  $a$  and  $b$
- (ii) By careful reasoning prove that  $a \leq b$
- (iii) Then note that by symmetry  $b \leq a$  hence
- (iv)  $a = b$

Mathematicians can see instantly here that because the initial labeling was arbitrary, the two objects are interchangeable (or symmetric) and hence the entire proof involving these objects is interchangeable (or symmetric) with the one where the labels are swapped.

Similarly, in geometry, the proof that the three medians of a triangle intersect usually starts by proving that two medians intersect. Since the two medians proven to intersect were not specially chosen, *by symmetry* any two pairs of medians intersect, and hence all three medians intersect.

The idea that symmetry allows abridged descriptions is of fundamental importance to our discussion of user interface design and affordance. In particular, symmetry if it applies to users' perceptions of a device allows them to be smaller and simpler. A user need only understand part of a system, provided it has the appropriate symmetries, to understand how to use all of it.

In physics, symmetry is extremely important as it is an essential feature of physics that the fundamental laws of physics are not particular but apply generally. For instance, there is an assumption that position in a particular frame of reference is not important: the laws of physics are invariant when translated from one place to another, say from the Earth to the Moon — that is, they have translation symmetry. Appropriate caveats need to be applied, as pointed out by Feynman [18], such as if a physics experiment is translated from a workbench into the adjacent wall (surely closer than the Moon) then we would expect the outcome to be somewhat different! Similarly, we anticipate

symmetries to have a deep role in user interface design, but nevertheless to be subject to caveats.

All of these symmetries clearly share common features, but exact formulations differ depending for each case. A general formulation due to Weyl [1] is:

A thing is symmetrical if there is something we can do to it so that after we have done it, it looks the same as it did before.

(We will develop this definition below.)

For example, a thing has a mirror visual symmetry if when we reflect it in a mirror it appears to be unchanged. A thing has a rotational symmetry if when we rotate it, it appears unchanged. Objects like balls have many symmetries, and objects like houses have fewer symmetries. For the examples above, it is clear that a transformation that exhibits symmetry can take many forms be it reflecting in a mirror, swapping over notation or considering the semantics of choosing and leaving. Correspondingly, what it means to “look the same” also varies from the actual process of visually perceiving to maintaining logical validity. Thus, symmetry is both a broad yet precise term and it is these features that make it useful.

#### 4. FORMALISING SYMMETRY

Whilst Weyl’s definition is adequate to understand the examples given above, a more careful definition is needed to ensure correct application in situations involving humans explicitly. This section therefore sets about providing a formal definition of symmetry with the explicit purpose of applying it to systems that people observe and interact with. We are thus building on the initial insight of [19, 20, 21] but making it more rigorous, general and useful.

A system,  $S$ , is some collection of states and attributes of various kinds and we can “look at” or perceive the system with a function  $\psi$ . For example,  $\psi(S)$  might be the perception that we see the buttons on the outside of the system, their labeling and their physical arrangement. In other words,  $\psi$  reduces  $S$  to some particular set of psychological attributes.

The system is typically complex and can have both external and internal changes made to it by transformations  $T$ . That is,  $T(S)$  is the same system having had some transformation done to it, be it moved from the table to the floor, pressing a button or receiving a signal from a GPS satellite. We now say, after Weyl, that  $S$  is symmetric if after applying a transformation to it, it seems to *the user* the same as it did before, or put symbolically,

$$\psi(S) = \psi(T(S))$$

Note that the system  $S$  here has a symmetry of type defined by  $T$ . For example, if  $T$  is a rotation, then  $S$  has a rotational symmetry if  $\psi(S) = \psi(T(S))$ ; in general we can say  $S$  has a  $T$ -symmetry or is  $T$ -invariant.

Weyl himself did not feel the need to introduce  $\psi$  in his discussion of symmetry, because he was not interested in what people *perceived* because of any symmetry. It is introduced here because perception is central to the nature of affordance; moreover, perception is not some simple bijection we can ignore at an appropriate level of formalisation, but is an imperfect process so that even if a system is transformed into a truly different state, this may still not be perceived by the user of the system. By having  $\psi$  explicit, it is possible to be clear about such effects.

As well as symmetries of the system, when we perceive an object, we may also identify symmetries in our perception. Here, it is not necessary to be explicit about perception, we assume that such symmetries are immediate to us and we see that certain psychological transformations,  $\tau$ , on the perceived system are indeed symmetries since:

$$\tau(\psi(S)) = \psi(S)$$

We deliberately leave the notion of perception without more detailed specification. This is because a rigorous definition would first require more detailed descriptions of the systems and minds under consideration, and exploring this then begs distinctions between cognition and perception. However we do note that there are many ways of perceiving an object. Individuals differ from each other in what they perceive and may even differ themselves at different times depending on what they have learnt between attempts to perceive the system; also their goals and what motivates them to perceive the system may change over time. Indeed, there is considerable research in the perception of symmetry, mental rotations of objects, and so on — for instance that people who practise rotating mental images of objects get better at it; see for example [22]. This is a fascinating area, but lies beyond the scope of the present paper.

Indeed, knowing and understanding the structure of  $\psi$  is an entire scientific enterprise; fortunately we do not need to “look in to”  $\psi$  for the purposes of this paper — this is one of the advantages of a formal approach. Even without reifying  $\psi$ , then, the notation already introduced is enough to make some (but not necessarily all) important issues explicit and hence start to draw sound arguments on how symmetry might apply to user interfaces.

#### 5. FROM PERCEPTIONS TO ACTIONS

The whole basis of affordance is that an object can be designed so that it has affordances. These affordances are identified by a person in some immediate and most likely unconscious way that leads them to recognise how they might interact with the object. The problem with affordances, as discussed above, is that it is not clear how the leap from attributes to interaction is made, nor is it clear how a designer can think clearly about these

issues in order to capitalise on affordance effectively in the design process.

In terms of our symmetry formalism above, there are several ways of representing how affordances might work. The simplest notation would be simply to say that some perception always leads to a particular set of actions,  $\alpha_i$ . This gives the sense of immediacy the affordances suggest:

$$\psi(S) \mapsto \{\alpha_i: i = 1, 2, \dots\}$$

An equivalent notation is occasionally more convenient:

$$\mathcal{A}[S] = \{\alpha_i: i = 1, 2, \dots\}$$

Here,  $\mathcal{A}[\ ]$  is the bundled-up psychological process that goes from the perception of the system  $\psi(S)$  to the set of actions. In particular,  $\mathcal{A}[\ ]$  is a mapping from the system, which the designer knows about, to possible actions, some of which the designer may know about and some of which may be entirely unique to particular individual users. For example, a person may perceive a pencil on a table and infer the action that it can be picked up and held in a writing position and designers would also know this. Some people (naughty children) may also see that it can be picked up and held in a position ideal for poking other people. For user interface design, we are obviously interested in the properties of  $\mathcal{A}[\ ]$  regardless of nuances of  $\psi$  that may occupy the concerns of professional psychologists.

It should be noted, though, that learning and experience can result in different perceptions,  $\psi'$ , but provided the perception is always equivalent, the deduced actions are always the same. Though this seems neat, there is the problem that learning may not actually result in different perceptions but rather that the different perceptions lead to different deductions about the set of actions.

We therefore need to be explicit about the model in which a person perceiving an object is reasoning in. More concretely, the perceiver of an object has some internal model,  $\mathbb{M}$ , a so-called user model, that constitutes their innate as well as learned abilities and knowledge of the system. Moreover, it is possible to reason in the model (possibly inaccurately) so that given a perception of an object, it is possible to deduce the actions,  $\alpha_i$ , that can be done to it. We write this formally as follows:

$$\mathbb{M} \models \psi(S) \mapsto \{\alpha_i: i = 1, 2, \dots\}$$

This, by the way, is the standard mathematical notation for this purpose [23]:  $\mathbb{M}$  is a model structure that gives meaning to the formula to the right of the  $\models$  sign. Our notation allows the user to have a non-standard interpretation of equality and other operations to the right of the  $\models$  sign, though we do not consider this in our current approach. Pedantically, to make this clear, we could write our equations as follows:

$$\mathbb{M}(\psi, \mapsto, \mathcal{A}[\ ]) \models \dots$$

This precision might be useful when, as below, we need operators like  $=$  and  $\Rightarrow$  (equality and implication): that is, we might want to distinguish whether the user *thinks*  $\Rightarrow$  or whether  $\Rightarrow$  is in some sense an objective fact. In this paper, we will avoid the clumsy notation because we are exploring affordance and symmetry, and it would take us too far away from this topic (and add no useful clarity) to consider the relation of mental models and logic (but see [24] for a classic discussion).

Now learning results in a change to the model ( $\mathbb{M} \mapsto \mathbb{M}'$ ) and this may result in a variant set of actions, even though the perception of the device is the same:

$$\mathbb{M}' \models \psi(S) \mapsto \{\alpha'_i: i = 1, 2, \dots\}$$

In this representation, it is enough to say that provided the model does not change, the same deduced actions will follow deterministically from the same perception of the system. Moreover, if the user perceives a distinct system,  $S'$ , the same way, then the same actions also follow. That is, if for some perception  $\psi(S) = \psi(S')$  then the model deduces the same actions:

$$\mathbb{M} \models \psi(S') \mapsto \{\alpha_i: i = 1, 2, \dots\}$$

For instance, if a person sees a friend's video player that looks identical to their own then they will assume that it works the same way. A deeper analysis of this would be to say that if a person perceives *part* of a system to be the same as *part* of some other system then those parts are immediately inferred to work the same way. It is possible to represent this by adding more structure to the perception formal model. However, a simpler way to do this is to say that the perception function is a restriction of a larger perception,  $\psi \sqsubseteq \Psi$ , but that, nonetheless, the same deductions follow in terms of actions that can be performed on that part of the system. Formally this requires the model to be monotonic.

The context of use should not alter the inference of possible actions provided the internal model,  $\mathbb{M}$ , of the perception is the same. Thus, if a person perceives a pencil, the actions inferred should be consistent regardless of the context in which it is perceived. However, some of the actions, such as using it for poking, may become more likely to be the action performed depending on the context.

As usual, there is a delicate trade-off between being clear in the formalism and being accurate to the complexity of real situations. At this stage, it would be premature to make the formalism more complex without first seeing whether we are making progress with the primary goals of clarity. In fact, we now have enough formal machinery to consider how symmetry can be used to understand user interfaces.

## 6. SYMMETRY IN USER INTERFACES

The first thing to consider is symmetries of the system itself. Suppose there is a system  $S$  such that some user with user model  $\mathbb{M}$  deduces some actions:

$$\mathbb{M} \models \psi(S) \mapsto \{\alpha_i: i = 1, 2, \dots\}$$

If this system is symmetric, there is (by definition) some transformation,  $T$ , such that:

$$\psi(S) = \psi(T(S))$$

However, the user's model is consistent in deducing actions from perceptions, hence:

$$\mathbb{M} \models \psi(T(S)) \mapsto \{\alpha_i: i = 1, 2, \dots\}$$

It is very important that our formalism can *completely* capture what we have said above partly in words and partly in mathematics. Indeed, we can write out the informal sentences and formulæ above in full formally:

$$\exists T: \mathbb{M} \models \psi(T(S)) = \psi(S) \Rightarrow \mathcal{A}[T(S)] = \mathcal{A}[S]$$

Note that the quantification of  $T$  is outside of the model  $\mathbb{M}$ . Thus, if there is a transformation  $T$  under which a system is symmetric then the inferred actions that can be done to the system are the same as before the transformation was done. In fact, the right hand side of the implication,

$$\mathcal{A}[T(S)] = \mathcal{A}[S]$$

is itself a symmetry.

Put another way, if a user does something to a system that does not seem to change it, they will believe they can still do the same actions to it as they could before. In a word, we might say that such actions are *afforded* by the system. However, to avoid confusion with the various nuances of the broad term *affordance* in the literature, we prefer to say that the actions are *symmetry-afforded* by the system.

Note that we have not made the trivial claim that  $T$  is a symmetry, so  $T(S) = S$ , and therefore  $\mathcal{A}[T(S)] = \mathcal{A}[S]$ . We have said that the user perceives a symmetry, namely that  $\psi(T(S)) = \psi(S)$ ; then we have said that in the model  $\mathbb{M}$  the user deduces sets of actions, and that these are the same.

For example, drawing a square here  $\square$  and another square here  $\square$  (or if you like,  $T(\square)$ ) are clearly drawings of different squares, but they *look* the same for most purposes, and it is an easy inference that we can do the same things with them.

Now consider two particular classes of transform of the system, namely, transformations in time and transformations in space.

A translation in time is simply that a system is in some initial state  $S_0$  and at some later point in time,  $t$ ,

it is in some other state  $S_t$ . If this particular temporal transformation is perceived as symmetric then:

$$\psi(S_0) = \psi(S_t)$$

and hence the user deduces the same set of actions  $\alpha$  at time 0 and time  $t$ . For example, if a person sees a light switch and infers that they can press it, temporal symmetry says that if they see the same switch at a later time, they will infer that they can press it in exactly the same way as before.

Spatial symmetry arises when a system  $S$  is moved between locations by a transform  $T_{xy}$  but provided it is perceived the same way, the affordance mechanisms infer that it can be acted on in the same way. That is,

$$\mathbb{M} \models \mathcal{A}[T_{xy}(S)] = \mathcal{A}[S]$$

Both of these ideas (translation in time and translation in space) together capture the notion of consistency. If a system is perceived in the same way at different times and different places, then the symmetry-affordances suggest the same set of actions on the system are (or should be) available. Of course, there is nothing more frustrating than this inference being incorrect, for instance when a battery dies (a broken temporal symmetry) or when a mobile phone fails to work when traveling abroad (a broken spatial symmetry). The *reason* why this is frustrating is that these are the basic Newtonian symmetries that we encounter all the time (and from the earliest age), and we have learnt to expect them more-or-less everywhere except when things break or decay.

Symmetries need not just be due to transformations of the system. Suppose a symmetry is actually (somehow) in the perception of the interface. For example, a person looks at a pencil and sees the rotational symmetry about the lead of the pencil. That is,  $\tau(\psi(S)) = \psi(S)$ . The inference then is:

$$\forall \tau: \mathbb{M} \models \tau(\psi(S)) \mapsto \{\alpha_i: i = 1, 2, \dots\}$$

That is, the user automatically infers that whatever actions were possible with the pencil in one position are true of the pencil in any rotated position (i.e., for all possible  $\tau$ ). In particular, if a pencil is perceived on a table and moreover the pencil is perceived as rotationally symmetrical, then the same action of picking it up to write with will be inferred as being effective. This particular symmetry-affordance is true for pencils, but false for fountain pens — the action of writing with a fountain pen also must incorporate an action to align the nib appropriately with the writing surface. From this, we infer that using fountain pens is harder than using pencils.

When it comes to affordances, our analysis with symmetry actually suggests two features of the psychological inferential mechanism that we have hitherto left undefined.

First, symmetries suggest actions. A symmetry in the perception of the system means that the user has actually perceived some property of the system and some corresponding transformation of the system that would leave the property unchanged. This immediately suggests that this could be incorporated into the inferential mechanism of deducing actions from perceptions, that is, the affordances of the interface. For example, if a user sees a circularly symmetric radio tuning knob, they perceive the symmetry and in particular that the transformation of turning the knob leaves the knob unchanged. Thus a specific action on the interface of the radio has become salient. Designers could make use of this to manipulate the affordance of user interfaces.

Secondly, and more subtly, symmetry-affordances suggest actions that *commute* with other actions.

Commuting and commutativity are mathematical terms. For example,  $+$  is an operator that commutes, since  $a + b = b + a$ , that is, the order of writing  $a$  or  $b$  first does not matter. In a user interface, moving a window and scrolling its contents are two operations that commute: they can be done in either order, with exactly the same result.<sup>3</sup> Opportunities for introducing commutativity are often overlooked in interaction design, even though, being a symmetry, they make user interfaces easier to use (because a user does not need to know the “right” order if all orders achieve the same result). The paper [25] introduces the term *permissiveness* — indicating that a user interface can *permit* different actions and different orders of actions to achieve the same goals. It is thus noteworthy that symmetry-affordance specifically highlights this design issue.

To see that symmetry-affordance suggests commutativity, consider a user applying an action that demonstrates the symmetry of their perception. As a result of this action on the system, their perception of the system is unchanged. That is, they see a rotationally symmetric radio knob, hence rotate it but the radio still looks the same. By the consistent nature of reasoning in their model, they still infer the same actions from their perception of the interface. If the system designer does not want to let the user down, those actions should behave exactly the same after the symmetric action was applied

<sup>3</sup>The use of the term commute technically raises philosophical issues — and philosophically raises technical issues. Algebra does not consider how a mathematician actually works; indeed, because of referential transparency it does not matter (from an algebraic point of view) whether in  $a \oplus b$  a mathematician works out the value of  $a$  or  $b$  first, they will get the same answer, regardless of whether the operator  $\oplus$  commutes. We note that  $a+b$  commutes for all numbers (numbers under  $+$  form an Abelian group), but in user interfaces “ $x$  then  $y$ ” only commutes for certain operations such as  $x = \text{move}$ ,  $y = \text{scroll}$  (in some user interfaces) — and if a user does “ $x$  then  $y$ ” this represents the fact that the user actually *has* done  $x$  first, even if “then” commutes in this case. For most user interfaces, user actions under “then” do not form an Abelian group (though a user interface may well be easier to use if they did).

as before. In other words, the symmetric action should commute with other actions. Again, in terms of the radio knob, turning down the volume should not affect your ability to tune the radio.

Both of these properties are not necessarily inherent in the psychological mechanisms of affordances but actually suggest that they should be. This, then, seems to be two concrete properties of symmetry that designers could exploit. Notice also, that to exploit these ideas does not require the full mathematical framework. The mathematics has revealed two ways in which symmetry either makes actions salient or requires a property of the user interface to hold. The only thing needed to address these affordances is to recognise and clearly define the symmetry.

## 7. UNDERSTANDING ASPECTS OF AFFORDANCE AS SYMMETRY

It is all very well claiming that symmetry adequately captures key notions of affordance, but it had better explain some of the existing examples of affordance — and perhaps some more.

We now consider some classic examples of affordance in turn, as well as providing some new insights on “common-sense” examples that hitherto have not been analysed in terms of affordance.

### 7.1. Staircases

Staircases have a translational symmetry (or translational and rotational symmetry if they are spiral). Thus if a person sees a staircase,  $S$ , they could perceive that there is a transformation,  $T$ , that maps stairs to adjacent stairs and hence  $\psi(T(S)) = \psi(S)$ . This makes the transformation  $T$  particularly salient. Of course, experience dictates that large objects like staircase are not easily transformed by actions of a person however, it is possible to bring about the perceptual transformation by moving up a step or down a step. Thus, the action of stepping is made particularly salient, that is, symmetry-afforded. Climbing or descending the stairs then follows by the symmetry as you can repeatedly climb up or down each step in turn and the action of stepping remains salient.

### 7.2. Door knobs, dials and door handles

Consider door knobs of the old fashioned variety that are simply large round brass knobs. These afford turning because of the symmetry-afforded action due to their rotational symmetry. Of course, turning is afforded equally in both directions but some door knobs do not live up to the affordance either because only a specific rotation actually unlatches the door or because they are broken so that rotation only has an effect in a specific direction.

With dials, such as tuning dials on a radio, a circular dial symmetry-affords a turning action. Again, whilst

the action is symmetry-afforded the consequences of the action are unclear. There are strong cultural conventions for how the dial should move as the knob is moved but these consequences are *not* symmetry-afforded [6].

What about door handles? In terms of symmetry, door handles do not have a particular symmetry and nor does the resulting motion of door opening. This is a case where symmetry *per se* does not explain affordance — this may be an affordance but it is not a symmetry-affordance. However, the lack of symmetry in the handle does better indicate that only a specific action will have an effect.

We can make the weaker claim that symmetry-affordance *does* explain the use of door handles if we allow that the user has learnt facts about the rotational behaviour of door handles. This is just another variant of temporal and spatial symmetries. The deeper relationship between the appearance of the handle and turning it may be due to inferred knowledge from the behaviour of objects fixed at only one end but couched in these terms, it is not a symmetric property intrinsic to the handle.

### 7.3. Buttons

As discussed earlier, it is not clear that a mouse can be used to move a pointer over the image of a real button and that clicking on the mouse would be the same as pressing the perceived button. However, if a user knows this then simply providing a button anywhere on the screen is a temporal and spatial symmetry for any particular button that the user remembers. Thus, by symmetry, if they can infer for one GUI button that it can be pressed, they can infer that all such buttons can be pressed. It would seem that given the indirectness of all such GUI interactions, all such affordances follow this temporal and spatial symmetry route.

As the discussion earlier showed, this sort of symmetry is better understood as consistency (things that look the same work the same way) and modelessness (this thing looks like it did earlier so it works like it did earlier). In that sense, the button itself has no symmetry-affordance other than those offered by every physical object.

### 7.4. Tangible interfaces

Tangible user interfaces are ones where there are physical objects that have virtual correlates that allow the user to interact with some information space. There can be many symmetries that work as affordances for the user.

For example, suppose a tangible interface is constituted from a set of identical cubes,  $\mathcal{C}_1 \dots \mathcal{C}_n$ , that can be placed on a flat two-dimensional, blank surface (for example, a tabletop),  $S$ . If the surface is sufficiently large then there are translational symmetries between one part of  $S$  and another, that is  $\tau(\psi(S)) = \psi(S)$

so if it is possible to place a cube at one point on the surface, it is possible to place the cube anywhere on the surface. Moreover, there is a symmetry between the cubes in that  $\psi(\mathcal{C}_i) = \psi(\mathcal{C}_j)$  hence it is possible to place any cube on the table and it should have the same effect as placing any other cube and all subsequent actions should also have the same effect regardless of which cube was placed. These are affordances that the system ought to satisfy as a result of the symmetries.

Additionally, the cubes themselves have symmetries. That is, they can be placed anyway up and in various orientations. This gives a whole set of transforms,  $T_1 \dots T_k$  such that  $\psi(T_k(\mathcal{C}_i)) = \psi(\mathcal{C}_i)$  and hence actions that makes these transformations become salient. That is rotating or rolling the cubes are symmetry-afforded.

Typically the operations on the physical objects will be connected to state transformations in some virtual representations of the objects. Obviously, if rolling the cube does something, there must be a state transition inside the tangible device. If so, that state transition of the device should respect the same symmetries of the physical cube. Specifically, the virtual representation should contain the symmetry group of the cube on a Euclidean plane (namely the group  $S_4 \times \mathbb{Z}_2 \times \mathbb{E}_2$ ) otherwise it will betray the natural physical symmetries the user expects of it from its physical affordances — the user might roll the cube and roll it back, but the system inside is not back in the same state.

### 7.5. Direct manipulation

Direct manipulation makes user interfaces easier to use. We can present this familiar, empirically-based claim in terms of symmetry-affordance.

There are many forms of direct manipulation; we will first consider a simple form. Icons can be clicked on and dragged to different parts of the screen. In some special parts of the screen, operations can be applied to icons, such as printing and deleting whatever the icons signify (such as file contents). Clearly the graphical user interface tracks for each icon its  $(x, y)$  coordinates. We know from physical objects that spatial translations generally leave the object the same, that is there are transformations that, with regards to icons, leave the icons unchanged. Thus, the actions that can be done to the icon should be independent of its location. This is what we find in the GUI: transforming the  $(x, y)$  coordinates leaves the icons unchanged. Thus, moving icons around is symmetry-afforded. However, when one icon is placed in the same location as another then that location is not like any other location and we can expect something different to happen. For example, colocating an icon for a document with a trash can icon is not like moving a document around on the desktop.

Once the user has mastered the conventions of moving a mouse and dragging icons, the user interface appears very natural — for it simply models the translation symmetries of two dimensional space,  $\mathbb{E}_2$ .

Indeed, many of the difficulties of learning to use a mouse occur when the symmetries break down, for instance at the edge of the screen or the edge of the mouse mat.

Some direct manipulation interfaces provide a zoom feature for improved accessibility: a user with poor eyesight can benefit from an enlarged visual image. Provided scaling the image changes no object properties, the accessibility feature does not change the other symmetries, and the direct manipulation interface remains as easy to use. Indeed, if the zoom is only visual, then zoom and translate commute and the user interface controls are unchanged; whereas if the zoom also scales mouse movement, then zoom and translate do not commute, and the user (with restricted mouse movement) has to use the interface differently to achieve the same effects.

### 7.6. Clocks

Marking the passage of time relies on the reuse of labels to make life much simpler. Thus, 5pm on 13th July gives a fairly specific time (the time at which this sentence is being written) but all of these labels are reused on a daily, monthly and annual basis respectively in order to make marking time a manageable process. And there is a good foundation for this system because the daily and annual cycles correspond to the rotational asymmetry of the Earth with respect to the sun.

Analogue clocks are generally circular reflecting the cyclical nature of counting time. However, as a consequence clocks have rotational symmetries, so ignoring hands, a clock,  $S$ , is such that after a rotation  $\tau$ ,  $\tau(\psi(S)) = \psi(S)$ . This tells us that after a rotation, the passage of time should still be marked out the same way. Or more concretely, the hands should move at a constant angular velocity. This also guarantees a correspondence between angles on the face and periods of time. Twenty minutes is always a third of the face regardless of which twenty minutes in the day they are. Likewise four hours. Additionally, because of the rotational invariance, operations such as putting on an alarm or setting the alarm time should be independent of the actual time shown on the clock.

Digital clocks, however, are notoriously hard to use, with no common standard of operation from one clock to another. Digital clocks are not significantly more complex than many other pushbutton devices, so why the complaint? Probably, we are unconsciously comparing the many elegant rotational symmetries of analogue clocks with the hidden and rather complex discrete-space symmetries of the digital clock. By contrast, digital clocks have numerous interdependencies. For example, you cannot increase the minutes beyond 59 in the same way you can increase them from 10 to 11; or if you can change the minutes beyond 59 the same way, the clock won't be able to run, because such times are invalid. Even accounting for 59

to 00 being the same gap as 10 to 11, such changes when setting the time can actually change the time by a whole hour rather than just a minute. Thus, there is no mapping between time periods and arithmetic differences the way that an analogue clock maps to spatial differences.

### 7.7. Breaking affordance

A well known concept is *symmetry breaking*, which happens then an object is modified so that its symmetries are reduced. For example, drawing a line across 0 results in  $\emptyset$ , which has fewer symmetries; the line breaks the symmetry. When a symmetry is broken, it may be possible to identify the cause of the broken symmetry, and perhaps rectify it.

Likewise, symmetry-affordance can be broken. The % key on different calculators looks the same, so in physical space it has translation symmetry. Yet % works differently on different calculators [26], and thus the expected  $\psi$  space affordance is broken. How is a user to interpret this broken affordance? Since calculators seem to be a given, and indeed a standard piece of modern life, it may be easier for the user to believe that their own particular  $\mathbb{M}$  is broken, rather than almost all calculators are broken. Indeed, this is what usually happens: users blame their own ignorance on their inability to predict what % keys should do.

The trouble with the informal term affordance is that it is easy to gloss over very sophisticated issues. For example, a balloon has various symmetries, yet to many people, the "affordance" of a balloon is to have a party, or to pop it, possibly both. There are no obvious ways to relate these sophisticated connotations to any perceptual or physical symmetries in the balloon, or indeed to any other basic psychological mechanism.

Given the confusion, how are we to choose the "right" symmetries to consider? In mathematics, the laws of symmetry are studied in group theory. A group is essentially system behaviour under all symmetries. This paper has implicitly drawn on group theory; the problem is that "groups" seem rather abstract to have any psychological validity. Further work is called for, and we suspect a fertile area will be in defining homomorphisms between the groups of different system representations: for example, between the physical user interface and the structure of the transition system "underneath" it.

There is also a mischief element in human nature that our examples can highlight as another form of affordance. Having had a particular action made salient by a symmetry-affordance, the mischief question then becomes, when can I abuse the symmetry-afforded action to break symmetry? For example, in the tangible interface example, the edge of the table is clearly distinct from the rest of the table, so what happens if you put a cube there? Wouldn't it be funky to have a clock that does not sweep out equal angles in the same

time periods? Would it be possible to have staircases that do not afford climbing? Indeed, this last question is the basis of tricks and deliberately designed challenges in games such as the *Tomb Raider* series where scenery is intended to be used for climbing even if it does not at first glance appear that way.

These questions go beyond the two principles of symmetry-affordance defined here into how other actions become salient as a consequence of symmetry and a mischievous nature. They may equally well be affordances in the more general sense but they would not be symmetry-affordances.

## 8. SUMMARY AND RECOMMENDATIONS FOR APPLYING SYMMETRY

To summarise, we claim that part of the psychological affordance inferential mechanisms (which we called symmetry-affordance) is that symmetries suggest actions that a user can do, and moreover that those actions respect the same symmetries. A designer can exploit this to ensure that users' expectations are met.

To apply the notion of symmetric-affordance in design, it is important to consider the key elements of the model. There are three key components:

- (i) The symmetries of the system, that is, transformations that leave the system looking the same;
- (ii) Symmetries in the perception of the system;
- (iii) The inferential mechanism that constitutes affordance.

The first point is the standard Weyl definition of symmetry. The second point arises because affordance requires consideration of the relation between physical objects and mental representations. We are not aware of the important role of the model being raised in any other discussion of symmetry. As has been seen though, an explicit inferential mechanism, even as lightly specified as the one here, is enough to provide insights on affordances. It is possible to consider deeper models that may base their inferences on things other than symmetries. As we have seen, learning and previous experiences can help to formulate the inferential model but exploiting these in a way as immediate as symmetry is not apparent to us.

We now summarise insights we have made in passing throughout this paper. (For brevity we do not revisit the mischievous and challenging potential — even “conventional” uses such as exploiting the security and educational potential of deliberate obscurity — of some design decisions, covered in Section 7.7.)

### 8.1. Insight 1: symmetries indicate actions

Symmetric parts of an interface, particularly a physical interface, suggest actions because of the immediacy of the symmetry and the transformation associated with the symmetry. Thus a symmetry in the interface

can indicate to the user's inference what actions are possible. A designer can exploit symmetry by ensuring that symmetric transformations really do correspond to meaningful actions on the system. A classic example of this is the “wheel” on an Apple iPod. Wheels have rotational symmetry, this suggests that the user could try turning the wheel and, fitting with this, turning the wheel does indeed help to control the interface.

### 8.2. Insight 2: broken symmetries indicate necessary feedback

Some transformations of the system need not be symmetric, for instance, a battery has died. In these cases, if the user does not perceive the difference caused by the transformation, they are going to infer actions that are potentially invalid given the new state of the system. At first glance, this may seem to lead to the well-known and well-worn usability guideline of providing the user with feedback [27]. However, with symmetry we are able to say more: if the system changes, but the inferred set of actions is not different then feedback on the system state may not be necessary. Thus, we can use symmetry as a way of deciding those transformations requiring attention (and so employ pop-out, colour changes and the like in the user interface) and those transformations that do not (and can simply result in a change of display that can safely be ignored).

### 8.3. Insight 3: symmetric actions should be permissive

Symmetrically prompted actions also have natural constraints. As discussed above, because of the consistency of inference of actions, symmetric actions should be permissive (commute with all other actions). Again, this need not be the case but in fact meeting with this expectation would support the symmetric affordance of the interface. Additionally, a symmetrically prompted action should always be effective as it is always cued for by the interface.

Interestingly, on the iPod, turning the wheel clockwise whilst at the bottom of a menu does not change the interface. Here, the symmetry-affordance has been broken and the resulting action is ineffective. There may be good reasons for this to do with scrolling through long lists but symmetry-affordance also suggests that the designers might not have got this right.

### 8.4. Insight 4: modelessness and consistency are symmetries

Modelessness and consistency are in fact specific cases of symmetry-affordance. They do not need dealing with separately but can be analysed, in the same way as other symmetry-affordances, by consideration of the underlying temporal and spatial translations.

### 8.5. Insight 5: the need for validation

Formal models stand or fall on their validity, which is a mathematical question. A proposed model of affordance further needs to be valid psychologically, and in some sense improve the quality of design or the quality of the design process. Unfortunately, validating symmetry-affordance in this sense strictly requires an appropriate experimental method, and access to designers with the necessary skills to generate testable designs based on the proposed concepts. This form of empirical validation is very much further work. In the meantime, our concept of symmetry-affordance is a research contribution that has value because it helps clarify an important concept, and raises many new questions. Whether such clarification leads to better designs is another matter; indeed, if it fails to lead to better designs, it is still possible that the formalisation was valid but the very concept of affordance is not as useful for HCI as we had hoped.

## 9. CONCLUSIONS

Affordance has seemed a very promising user interface design concept, yet one that has been hard to pin down. People have used the term very freely, and people have used it in very different ways, and the original what-might-have-been tight definition has been lost. However it is not necessary to throw out the promise along with rejecting the vagueness.

There is controversy whether affordances are learnt, cultural, genetic, or physiological, etc. From our point of view, though, we wanted to formalise affordance so it could be used constructively in user interface design. From our point of view, then, this paper brought new clarity on the following point: if it wasn't for symmetry-affordance, users would *have* to keep re-learning user interfaces, because their laws of interaction would keep changing. Symmetry-affordance is precisely the point that how a device interacts under certain user actions does not change. Affordance makes things easier to learn to use and easier to know how to use from past experience. This point is most clearly made by Weinberg [28], though from the point of view of a physicist understanding reality, rather than from the point of view of a user understanding an interactive system.

We developed a theory of symmetry to underpin a new approach to affordance, namely symmetry-affordance. Our theory of symmetry extends conventional discussions of symmetry in a crucial way by making the user model and perception explicit.

Symmetry-affordance clearly captures some important notions of usability; we have given some reasons why this may be the case, and we have given some examples that show it "in action."

There are many exciting strands of future work opened up.

First is addressing this paper's almost-hypocritical jump from formality to speculative examples and illustrations of symmetry-affordance. Our initial position was that the use of the term affordance in the wider literature was diverse and informal when it might be more precise, but we then failed to carry through our programme of reform even with our own examples. We see this, rather, as an opportunity for more research, and that our outlined examples show both the value and plausibility of such research. We expect, of course, that such research would refine our formalisation, and might even expose contradictions or serious limitations in our approach. Of course, such advances can only occur because we dared to be sufficiently rigorous (at least in the main contributions of this paper) that omissions or flaws in our approach might become obvious.

The recommendations listed in this paper we believe would help designers design better systems — and not just because they encourage designers to consider more options! However, the recommendations and insights themselves beg research questions, that is, whether they are sound. For example, systems might be built contrary to some recommendation yet might be found to be better for certain tasks — there are very many interesting avenues of research beyond the scope of this initial paper. If symmetry-affordance survives empirical research, then further questions are raised. How can we establish the relevant symmetries of user design requirements? How can we build interactive systems that respect desired symmetries, and in particular, how can we capture such requirements in a way that makes sense at the abstract level of programming?

Symmetry-affordance as a concept itself also begs a psychological research question: namely, just how does the brain "resonate" (to use Gibson's word) to detect the symmetries?

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