“B–” for *The da Vinci Code*

Harold Thimbleby
Department of Computer Science
Swansea University

h.thimbleby@swansea.ac.uk
http://www.cs.swan.ac.uk/~csharold

Dan Brown’s *The da Vinci Code* has become a publishing phenomenon, not least because of its inaccurate and controversial recasting of early Christian history that upset the Church. Its science is inaccurate too. We might allow the author creative freedom to revise history to get a good story, but there seems no excuse for mangling the science, especially when getting it right would have made the story more plausible. *The da Vinci Code* isn’t science fiction; it pretends to be fact, like a historical novel where the characters are fiction, but the historical context is taken to be true. The book even has a page explicitly saying the substantial background is fact — although that page is part of the novel’s revisionist story.

Dan Brown puts silly words into the mouths of his characters, who we are expected to believe are experts: a Harvard professor, a graduate of the Royal Holloway College’s leading security course, and a top historian. Even *The da Vinci Code’s* facts about bee keeping are wrong.

The inaccrate bee keeping nicely illustrates how the page-turning imperative of this blockbuster thriller knocks truth and accuracy into second place throughout the book. The author — or, rather, the words he puts into characters who should know better — mangles basic facts, and presents confusing inaccuracies instead of the truth. Often, the truth of the issue is much more interesting than the garbled version. The book is quite exciting, with police chases, loose murderers, and mysterious clues, but the tension often seems used to to undermine the reader’s critical faculties: the worst mangles happen just when the narrative urge to keep reading and turning pages is overwhelming!

The story starts off with a nasty murder in the Paris Louvre. The victim managed to scrawl some jumbled up Fibonacci numbers on the floor next to his naked body. Perhaps he has written a clue to who is murderer was? The Fibonacci numbers, 1 1 2 3 5 18…, are written down in a mysterious order as 13-3-2-21-1-1-8-5. A Harvard professor, Robert Langdon, is a key suspect. Possibly the numbers are there to stage the entrance of Sophie Neveu, a cryptographer who recognises them for what they are. Once the Fibonacci numbers have been explained to the professor, the story gets going.

Later, Neveu explains to Langdon about a thing she calls a cryptex. She suggests Leonardo da Vinci invented something called a cryptex as an elementary form of public key encryption. None of this is true. If the murder victim had wanted to use public key encryption, he could have done easily, and a simple email could have given his secret more security than the imaginary cryptex.

Another example is when we hear that “Venus traced a *perfect* pentacle across the ecliptic sky every eight years.” It’s not obvious what this means, though it could be based on the fact that about eight times every five years, Earth and Venus are more-or-less in a straight line with the Sun (when they are in conjunction), and if you draw lines between where they are in orbit at each conjunction, you get an approximate pentagram, and if you draw in a circular orbit as well, you get an approximate pentacle (a circle enscribed round the pentagram). You cannot see this imaginary pentacle from Earth, and certainly not by looking in the ecliptic plane. So, there is an interesting pentacle in the story of Venus, but not where or when the book suggests. Indeed, the ratio of 5:8 is a ratio of Fibonacci numbers, which we've already seen play an important part in the book — but this isn't mentioned.

All this sounds like quibbling, and perhaps it is. The moral is that a reader of a novel should remain cynical — though that's hard when the story is exciting. Even under pressure, a scientist should check their facts.

Soon the Harvard professor is daydreaming about a lecture he gave to his students on the divine proportion, which he calls PHI. In fact it is more widely written as ϕ, the Greek letter of that name, and it's more commonly known as the Golden Ratio — there's nothing especially divine about it, unless you are writing a theological consipracy thriller! The value of ϕ is 1.618, approximately. It is an irrational number, the value of $\frac{1+\sqrt{5}}{2}$ (as we'll see below). As such, ϕ has no exact numerical value, despite *The da Vinci Code*'s excitement about many amazing things being *exactly* equal to ϕ — which is simply not possible!

The Golden Ratio turns up for good reason in many surprising places because it is the solution of a simple and common equation. It turns up in bee hives, which is a point Dan Brown tries to make at this point in the story. But he gets his facts wrong; and in getting them wrong he misses out on a much more interesting and deeper story — which we'll explore in a moment.

First, let's pick up the story, where Langdon talks about bees. In the thriller we get the following stream of thought and dialogue:

> Plants, animals and even human beings all possessed dimensional properties that adhered with eerie exactitude to the ratio of PHI to 1. …
>
> So the ancients assumed PHI must have been preordained by the Creator…
>
> Langdon grinned. "Ever study the relationship between female and males in a honeybee community?"
>
> "Sure. The female bees always outnumber the male bees."
>
> "And did you know that if you divide the number of female bees by the number of male bees in any beehive in the world, you always get the same number?"
>
> "You do?"
>
> "Yup. PHI."
>
> The girl gaped. "No way!"

Page 132

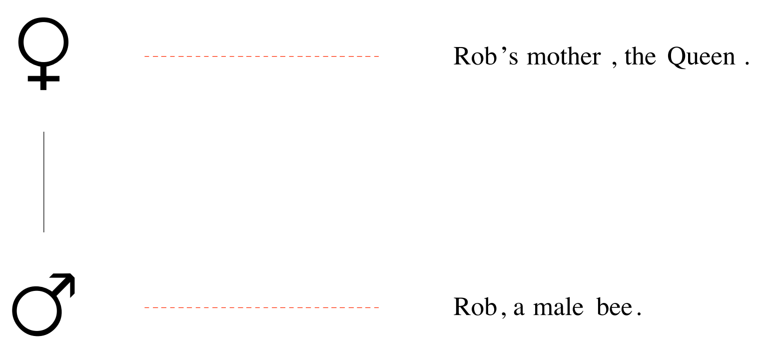
This couldn't be more wrong, especially the so-called *eerie exactitude*! Approximately 95% of a bee hive will be workers, and the workers are female, so that leaves 5% males. If you divide 95 by 5 you get 19, which is nothing like 1.62, and it's nothing that could be called an eerie exactitude as PHI to 1 is 1.618, not 19. In severe winter, all the male bees die off, so then the ratio is infinite! At first sight, then, the facts make Dan Brown look completely wrong. But did Dan Brown hear something interesting about bees and the Golden Ratio that he has lost in the retelling?

On the next page of the book, Langdon is claiming that the human body is made of building blocks whose proportion is *always* equal to PHI. Langdon tells his students to take a tape measure into the shower to check his claims. Like any good scientist would, I checked — and he's wrong, though I did come up with some other very interesting numbers that I'm sure must have mystical significance in another story. Similarly, it's worth checking out his claims about bees.

Bees are very interesting. A male bee, a drone, hatches from an unfertilised egg, whereas all females hatch from fertilised eggs. There are further complications in that some females are queens and some workers. Workers are sterile females, and usually have no offspring. In a hive there is normally one queen, busy laying maybe 2000 eggs per day. From time to time, there are females who are fed royal jelly and they grow into a new queens.

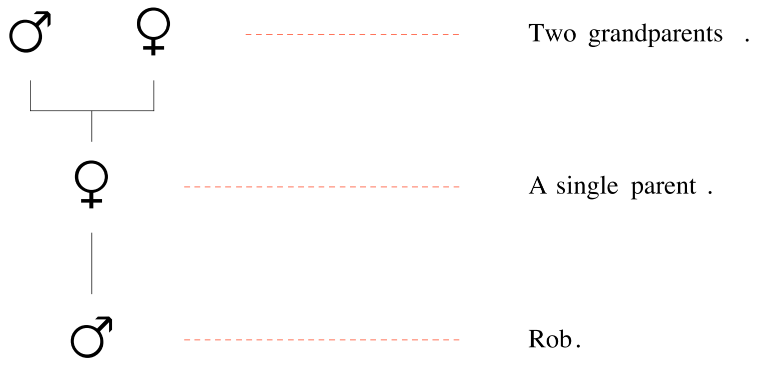
In a typical honey bee hive that's going well there will be tens of thousands of workers. In nature, when a successful hive gets too big, one or more queens will buzz off with a swarm of bees to start new hives somewhere else. This behaviour would be disastrous for a bee-keeper, as they'd loose most of their bees, so well-managed bees are usually fooled into not swarming.

Let's try and work out what the facts about the female to male ratio are, and where Dan Brown might have got his ideas from. It's convenient to represent males and females by their usual signs, ♂ for males and ♀ for females. Now imagine a single male (♂), who we'll call Rob. Now Rob, being a male, has one parent, the queen (being female, we'll show as ♀). We can draw a family tree for this simple single parent family:

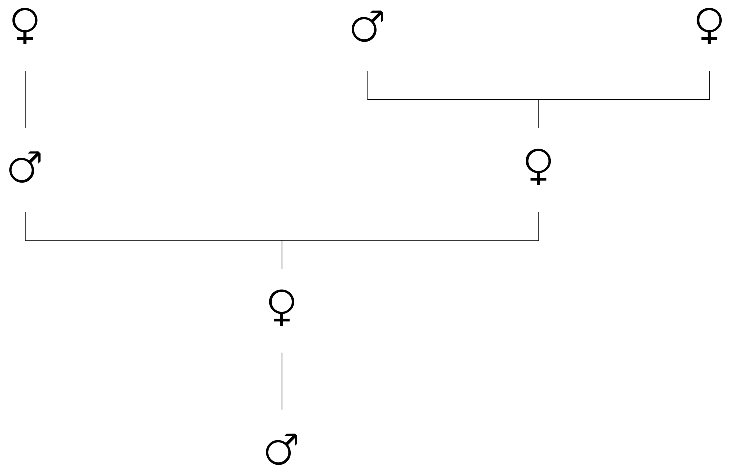


In turn, the queen here, being female, herself had two parents.

Now, let's expand the family tree further back to include Rob's grandparents:



Rob has one parent (the queen) and two grandparents (a drone and another queen, probably a queen from a hive far, far away). In turn these two grandparents will have their own parents, who will be Rob's great grand parents. Again, we can draw a family tree:



Looking at Rob's family tree, count the number of bees in each generation. Start with the original bee, Rob, who has 1 parent. Rob has 2 grandparents, then 3 great grandparents. And, going beyond the drawing, then there will be 5 great great grandparents, 8 great great grandparents, 13 great great great grandparents. You can easily draw the family trees to check. Do you notice that these numbers are the same Fibonacci numbers 1 1 2 3 5 8 13… we saw earlier in the story? Indeed, there will be 21 great great great great grandparents.

These Fibonacci numbers are in order; they aren't jumbled up, like they were in the murder victim's final message. What *was* the murder victim thinking about?

Fibonacci numbers are easier to work out than counting Rob's ancestors. Each number in a Fibonacci sequence is the sum of the previous two. After the first two ones, the next is 2, which is 1+1. After 2 comes 3, which is 1+2, adding the two numbers preceding it. After 3 comes 5, which is 2+3. Then 8, which is 3+5. In general, if two consecutive Fibonacci numbers are x and y, the next one is $x+y$.

Curiously Fibonacci, the twelth century mathematician who invented Fibonacci numbers, originally got them through thinking about breeding rabbits, not bees! In fact, Fibonacci's rabbits were rather contrived to get the numbers to come out right, whereas our bees do what real bees do and the numbers work out just right. Perhaps Fibonacci did not know enough about bees? Although we now remember Fibonacci because of his numbers, he should be remembered as well as the person who brought Arabic numbers to Europe — eventually which replaced the ungainly Roman numerals that were used everywhere in those days.

So we've certainly got a mathematical connection back from this story of bees to the earliest pages of *The da Vinci Code*, but what about the female-to-male ratio that Professor Langdon mentioned? What have these numbers got to do with ϕ, the Golden Ratio?

The Golden Ratio comes back in because the ratio of consecutive Fibonacci numbers get closer and closer to it.

You can work out 1/1=1, 2/1=2, 3/2=1.5, 5/3=1.67, 8/5=1.6 and imagine they do get closer and closer to 1.618. Are you sure?

It is more convincing to get a computer program (or use a calculator) to work out the ratios a bit further:

N	Nth Fibonacci number	Ratio of Nth and N−1th Fibonaccis
1	1	
2	1	1.00000000
3	2	2.00000000
4	3	1.50000000
5	5	1.66666667
6	8	1.60000000
7	13	1.62500000
8	21	1.61538462
9	34	1.61904762
10	55	1.61764706
11	89	1.61818182
12	144	1.61797753
13	233	1.61805556
14	377	1.61802575
15	610	1.61803714

So, the ratios certainly look like they get closer and closer to a number near 1.6180…. Is this the Golden Ratio?

Can we work out *exactly* what number they converge to? Take a Fibonacci number a long way into the sequence, say the millionth or billionth, and call its value x. If the ratio of consecutive numbers is converging to the Golden Ratio, ϕ, then the next number in the sequence should be practically ϕ times x. The next number will again be ϕ times it, or ϕ^2 times x. Ah, but we *also* know that as a Fibonacci number this third one, $\phi^2 x$, must be the sum of the two numbers preceding it, which are x and ϕx. So we can write down an equation:

$$x + \phi x = \phi^2 x$$

If you are an advanced mathematician, you can be more precise about what 'practically' ϕ times x really means, but a more rigorous argument turns out the same in the end. Having got this equation, we can divide both sides by x and rearrange it to $\phi^2 - \phi - 1 = 0$. This is a standard quadratic equation, and there is a formula for solving such equations. The formula gives us

$$\phi = \frac{1+\sqrt{5}}{2} \quad \text{or} \quad \phi = \frac{1-\sqrt{5}}{2}$$

These solutions give two possible values for ϕ, one of which is 1.618033988749894848…, which is what we were expecting. This is the Golden Ratio.

You may be wondering what that 'other' value for ϕ is, the other solution to this equation. It' s –0.618. It's an interesting number in its own right: for example, if you subtract 1 from it you get $-\phi$.

Usually when we have a quadratic, both of the solutions mean something, so it's worth spending a moment to work out what this other solution might mean. We got $\phi = 1.618$ as the ratio of consecutive Fibonacci numbers as we moved along going 'to the right' in the sequence — so the obvious question to ask is, what happens if we go in the other direction, to the left?

Think back to the first two Fibonacci numbers, 1 and 1. We know the *next* Fibonacci number after them is 2 because 1+2=2, but what number could *precede* them? It has to be zero, since 0+1=1. Now we can go back one more to the 'left' along the sequence. Before zero, what? It has to be the number that added to 0 gives 1. That's 1.

Here's what we've worked out so far, starting to extend the Fibonacci sequence to the left:

… 1 0 1 1 2 3 5 8 …

And before that 1, there must be a number that added to 1 gives zero. That's –1.

… –1 1 0 1 1 2 3 5 8 …

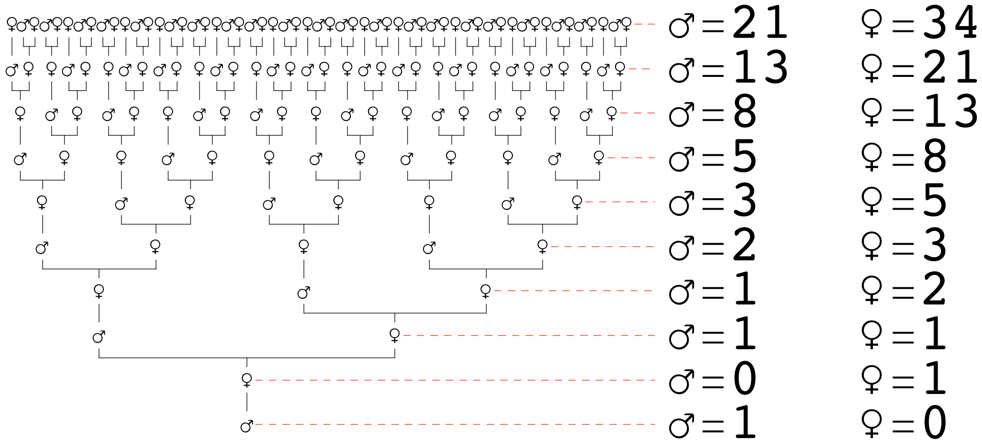
And before *that* –1, there must be a number that added to it gives 1. That would be 2, of course. And if we carry on like this we get the Fibonacci sequence again, going back the other way. The Fibonacci numbers going this way have alternating signs:

… –21 13 –8 5 –3 2 –1 1 0 1 1 2 3 5 8 13 21 …

If the numbers alternate positive and negative, their ratios must be negative. As the numbers are getting bigger in the opposite direction, now the ratio has a smaller number divided by a larger number: the ratio now must be less than 1. So that's where –0.618 comes from; it's the ratio of consecutive Fibonacci numbers going towards infinity in the other direction.

Now we know how bees relate to Fibonacci numbers, and how Fibonacci numbers relate to the Golden Ratio, let's go back to Rob's family tree.

Let's draw a family tree covering more bee generations, counting off for each generation the numbers of males and females:



The numbers of males and females taken *separately* are Fibonacci numbers! In fact, for each generation they are *consecutive* Fibonacci numbers, such as 2 and 3, or 8 and 13.

We should check: are we quite sure? Consider some generation where there are N_f females and N_m males. Well, these bees will have had a total of $N_m + 2N_f$ parents, specifically $N_m + N_f$ females and N_f males. So, in any generation in the family tree, you can see the number of females is the sum of the number of male and female offspring, whereas the number of males is the exactly the number of female offspring. Put another way, if the numbers of males and females in one generation are N_m and N_f, that generation's parents will have N_f and $N_m + N_f$ males and females. Do you recognise the number of females is the sum of the males and females in their offspring? Each generation of bees simply generates Fibonacci numbers.

Provided the sums start off with 1 and 1 (Rob and his parent), then we will generate the Fibonacci numbers in each generation. Now we know the ratio of female bees to male bees will be the ratio of consecutive Fibonacci numbers, which, as we've seen, will converge on the Golden Ratio.

So it looks like the mysterious fact Dan Brown mentions about the ratio of female and male bees in a hive is not the ratio *in a beehive*, but the ratio of sexes *in any bee's ancestral tree*. The more generations back you go, the better the ratio of females to males ancestors for any particular bee will approach 1.618.

We've got a good idea how Dan Brown got his inspiration for the beautiful idea that Professor Langdon misunderstood. However, the full story isn't as simple as this. There's a problem! The further back in the family tree we go, each generation must have more and more bees: 1, 1, 2, 3, 5, 8 and so on. Fibonacci numbers get bigger and bigger.

If we go back far enough in Rob's ancestral tree, there must have been an absolutely *huge* number of bees. The further back one goes in the family tree, the more bees seem to be needed. Unfortunately, if we go far enough back, the huge numbers of bees involved begins to look unrealistic. Far enough back and we'd need more bees than there is space on the Earth to house them! In other words, something is wrong with our story. This problem occurs with any sexually breeding animals, including humans, and is called the ancestor paradox.

One explanation is that life originated in outer space: that's where the bees *must* have come from if the Earth isn't big enough. While a quirky idea, some people believe life originated in outer space. Unfortunately, Fibonacci numbers get bigger and bigger without limit, and even outer space would not be big enough. So this alien theory isn't very helpful (or, to be fair to it, it can't be the only explanation)!

There are more plausible explanations. Some of the bees we counted may be being double-counted: queens lay lots of eggs, and some of the females we counted as separate individuals may in fact be one and the same individual — and if so, their parents will be the same, and they will have been double-counted too. This is *inbreeding*. (We won't have been double-counting males, since, sadly, when they copulate with a queen to fertilise it, mating rips their reproductive organ and other tissues out of their bodies, so they don't live to mate again.)

Some bees might appear, double-counted, in the family tree in different generations, and this is *linebreeding*.

In fact, inbreeding will be common. When an old queen takes flight in a swarm to start a new hive, she will be followed closely by her own drones, so usually she will be fertilised by her own offspring. When a new queen takes flight, she will typically be fertilised by her brothers, except when the hive she is leaving had more than one egg-laying female (which in any case would most likely be her sisters). Each occasion of inbreeding halves the size of the generations properly counted in the ancestral tree, and since we were assuming the sizes of the parent generations were growing at the rate of 1.618 as we go back in time, it's clear that not much inbreeding is required to avoid the paradox altogether, since each bit of inbreeding divides the 1.618 by 2.

Or our biology may be faulty in other ways too. Indeed, we know that if we go back far enough in time that bees must have evolved from something else, and it's unlikely that evolutionary ancestor had such peculiar sexing rules. Another possibility is that parasites and disease alter the genders of bees, though this in itself does not solve the numbers paradox. However we avoid the ancestor paradox for the bees, it's inevitable that Dan Brown will be wrong again, since the ratios of males to females won't be exactly the Golden Ratio.

To summarise where we've got to: we now know that the ratio of ancestor female and males will be *nearly* ϕ, but it can never be *exactly* ϕ, because bees don't quite follow the Fibonacci rules. Also, we noted that the ratio of female to male bees in a single hive is around 20, nothing like the Golden Ratio.

The da Vinci Code is a page-turner, and the bees only take up a page of its story. So what, then, about being precise about Fibonacci numbers and the Golden Ratio? First, the book gives its readers the impression it is being accurate; there's even a page that says as much. The faulty discussion of bees is put in the mouth of a Harvard professor, who ought to know his facts. Secondly, the real facts about bees ties up with the story's use of Fibonacci numbers very well, so it's a lead Brown missed. And thirdly, the true story of bees is more interesting than the book suggests. In fact, this is the problem with the book: it mangles facts, often quite unnecessarily, when the truth is more interesting. Anybody who wanted to pass an exam in maths, astronomy, bee-keeping or early Christian history would be better off revising from a book that knows what it's talking about. It's arguable that some of the contrivances in his story set up the exciting framework to carry the thriller along, but there is no reason to mangle the science.

Brown's book starts with a big banner saying *Fact: … all the descriptions of artwork, architecture, documents and secret rituals in this novel are accurate*. No way. Perhaps we can look forward to a blockbuster of a book that starts off by saying *Fact: all the descriptions of artwork, architecture, science and maths, documents and secret rituals in this novel are accurate*. Ah, the secret ritual of a scientist is to check facts, and, as we saw, in turn to check the implications of the facts we check. We learn a lot about bees if we do, and we realise there's still a lot more to discover and check. And so science weaves its story.

- **D. Brown, *The Da Vinci Code*, Corgi Books, 2003.**

- **M. Haag & V. Haag, *The Rough Guide to The Da Vinci Code: History, Legends, Locations*, Rough Guides (Penguin), 2004.** This book thoroughly explores the background to *The da Vinci Code*, and exposes many of its errors, from Gnostic gospels to art history.

Bibliography

Prof Harold Thimbleby is professor of computer science at Swansea University Wales, and directs the Future Interaction Technology Laboratory, www.fitlab.eu. He is a Royal-Society Wolfson Research Merit Award Holder, and was recently the Gresham Professor of Geometry — holding a chair whose history goes back to 1597 with Henry Briggs, the inventor of natural logarithms. His own web site is at www.cs.swan.ac.uk/~csharold

This article is available at http://www.cs.swansea.ac.uk/~csharold/dvc as a PDF file. It's also available as a *Mathematica* document, which (if you have *Mathematica*) you can run and draw all the pictures and check all the sums.